

Multi-Digit Multiplication Strategies

4th Grade

Mathematics Formative Assessment Lesson

Designed and revised by Kentucky Department of Education Mathematics Specialists
Field-tested by Kentucky Mathematics Leadership Network Teachers

If you encounter errors or other issues, please contact the KDE team at:
kdemath@education.ky.gov

Created for the sole purpose of assisting teachers as they develop student understanding of Kentucky's Core Academic Standard through the use of highly effective teaching and learning.

Not intended for sale.

Multi-Digit Multiplication Strategies

4th Grade

This Formative Assessment Lesson is designed to be part of an instructional unit. This task should be implemented approximately two-thirds of the way through the instructional unit. The results of this task should then be used to inform the instruction that will take place for the remainder of your unit.

Mathematical goals

This concept-based lesson is intended to help you assess how well students are able to use a variety of strategies to multiply. In particular, this unit aims to identify and help students who have difficulties with:

- The traditional multiplication algorithm.
- Representing multiplication in multiple ways.

Common Core State Standards

This lesson involves a range of *mathematical practices* from the standards, with emphasis on:

- 2. Reason abstractly and quantitatively.**
- 7. Look for and make use of structure.**
- 8. Look for and make use of repeated reasoning.**

This lesson asks students to select and apply mathematical content from across the grades, including the *content standards*:

Number and Operations in Base Ten

4-NBT: Use place value understanding and properties of operations to perform multi-digit arithmetic.

5-NBT: Perform operations with multi-digit whole numbers.

Introduction

This lesson is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. You then review their work, and formulate questions for students to answer, to help them improve their solutions.
- During the lesson, students work in pairs and threes to match the word problem, model, and multiple strategies of the same multiplication or division problem.
- In a whole-class discussion, explain their answers.
- Finally, students return to their original assessment task, and try to improve their own responses.

Materials required

Each individual student will need two copies of the worksheet *Multiplication Strategies & Representations*.

- Each small group of students will need a packet of Card Set A - F copied in **color** cut up before the lesson. *{Note: you may want to make color copies, and laminate these for use in multiple classes over multiple years.}*

Time needed

Approximately fifteen minutes for the assessment task, a one-hour lesson, and 15 minutes for the students to review their work for changes. All timings are approximate. Exact timings will depend on the needs of the class.

Before the lesson

Assessment task:

Have the students do this task in class a day or more before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. Then you will be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Multiplication Strategies & Representations*. Introduce the task briefly and help the class to understand the problem and its context.

Spend fifteen minutes on your own, answering these questions.

Don't worry if you can't figure it out.

There will be a lesson on this material [tomorrow] that will help you improve your work.

Your goal is to be able to answer these questions with confidence by the end of that lesson.

Mathematics Formative Assessment Lesson		Student Materials		Revised Alpha Version	
Multiplication Strategies & Representations Task					
1.) Multiply 28 by 17 and show your work:					
2.) Sam, Julie, Pete, Lisa, & Fred each multiplied 28 by 17. Below each method indicate if the work is correct and then explain whether that method makes sense mathematically or not.					
Sam	Julie	Pete	Lisa	Fred	
$\begin{array}{r} 28 \\ 17 \\ \hline 196 \\ + 280 \\ \hline 476 \end{array}$		$\begin{aligned} 28 \times 10 &= 280 \\ 28 \times 5 &= 140 \\ 28 \times 2 &= 56 \\ 280 + 140 + 56 &= 476 \end{aligned}$	$\begin{array}{r} 28 \\ 17 \\ \hline 200 \\ 140 \\ 56 \\ \hline 476 \end{array}$		
Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	
3.) Which method most closely matches how you solved the original problem? _____ Choose a <u>different</u> method than what you used in #1 to multiply 35 by 14. Show your work below:					

It is important that students answer the question without assistance, as far as possible. If students are struggling to get started, ask them questions that help them understand what is required, but do not do the task for them.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and their different problem solving approaches. The purpose of

this is to forewarn you of the issues that will arise during the lesson, so that you may prepare carefully.

We suggest that you do not score students' work. The research shows that this is counterproductive, as it encourages students to compare scores, and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of their work. Some suggestions for these are given on the next page. These have been drawn from common difficulties anticipated.

We suggest that you write your own lists of questions, based on your own students' work, using the ideas below. You may choose to write questions on each student's work. If you do not have time to do this, select a few questions that will be of help to the majority of students. These can be written on the board at the beginning of the lesson.

Common issues: Suggested questions and prompts:

Common Issues	Suggested questions and prompts
Student doesn't match the cards correctly because he or she doesn't have a conceptual understanding of multiplication.	<ul style="list-style-type: none"> <i>If you are multiplying 27×4, what does the 2 represent? the 7?</i> <i>What would happen if you multiplied 20×4 and 7×4? Could you use those answers and to get the answer to 27×4?</i>
Student doesn't understand Distributive Property.	<ul style="list-style-type: none"> <i>How can these number(s) we are multiplying be broken apart?</i> <i>What could you do with <u>those</u> numbers to solve this problem?</i>
Student doesn't understand the area model for multiplication.	<ul style="list-style-type: none"> <i>In the problem 27×14 let's look at the number 27. How many 10s are in 27? How many ones? How could you model 27? Now let's look at 14? How many tens? ones? How could you model 14?</i> <i>Is there a way to take those two models and fit them on a rectangle to discover 27×14 without doing any calculations?</i>
Student doesn't understand the lattice method for multiplication.	<ul style="list-style-type: none"> <i>How do the numbers in the lattice grid compare to the numbers in the partial products and traditional methods?</i> <i>How can use the numbers along each diagonal in the lattice grid to get your final product?</i>

Suggested lesson outline

Collaborative Activity: matching *Card Sets Models A, B, C, D, E* and *F* (30 minutes)

Organize the class into groups of two or three students. With larger groups, some students may not fully engage in the task.

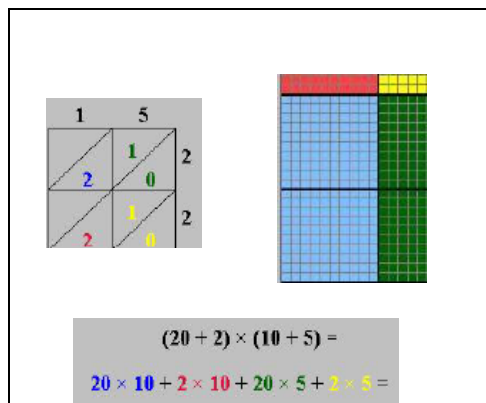
Give each group *Card Sets A, B, and C* – area model, lattice & distributive property.

Introduce the lesson carefully:

I want you to work as a team. Take it in turns to match a Model card with either a Lattice card or a Distributive Property card.

Each time you do this, explain your thinking clearly and carefully. If your partner disagrees with the placement of a card, then challenge him/her. It is important that you both understand the math for all the placements.

There is a lot of work to do today, and it doesn't matter if you don't all finish. The important thing is to learn something new, so take your time.



As the teacher, your tasks during the small group work are to make a note of student approaches to the task, and to support student problem solving

You can then use this information to focus a whole-class discussion towards the end of the lesson. In particular, notice any common mistakes. For example, students may know Lattice multiplication and the Distributive Property but may not understand the model.

Make a note of student approaches to the task

Try not to make suggestions that move students towards a particular approach to this task. Instead, ask questions to help students clarify their thinking. Encourage students to use each other as a resource for learning.

Students will correct their own errors once the Partial Product and Traditional Algorithm cards are added.

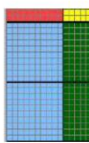

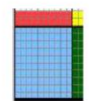

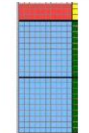

For students struggling to get started:

There is more than one way to tackle this task.

Can you think what one of them might be? [Working out the answer in either the Distributive Property problem or the Lattice multiplication model and matching that card to the model or finding the answer from the model and matching it to either the Lattice or Distribution Property card.]

How can you calculate products with the Distributive Property? with the Lattice model?

This Distributive Property card shows $(20 + 2) \times (10 + 5)$? What would the original multiplication problem be for this model? Does that multiplication problem match any of the other cards on the table?

Multiplication Representations & Strategies CARDS					
Model Card Set A	Lattice Card Set B	Distributive Property Card Set C	Partial Products Card Set D	Traditional Card Set E	Problem Card Set F
		$(20 + 2) \times (10 + 5) =$ $20 \times 10 + 2 \times 10 + 20 \times 5 + 2 \times 5 =$	$\begin{array}{r} 15 \\ \times 22 \\ \hline 30 \\ 300 \\ \hline 330 \end{array}$	$\begin{array}{r} 15 \\ \times 22 \\ \hline 30 \\ 300 \\ \hline 330 \end{array}$	Each pack of baseball cards has fifteen cards. How many cards are in twenty-two packs?
		$(10 + 2) \times (10 + 2) =$ $10 \times 10 + 2 \times 10 + 10 \times 2 + 2 \times 2 =$	$\begin{array}{r} 12 \\ \times 12 \\ \hline 24 \\ 120 \\ \hline 144 \end{array}$	$\begin{array}{r} 12 \\ \times 12 \\ \hline 24 \\ 120 \\ \hline 144 \end{array}$	How many eggs are in twelve dozen?
		$(20 + 3) \times (10 + 1) =$ $20 \times 10 + 3 \times 10 + 20 \times 1 + 3 \times 1 =$	$\begin{array}{r} 11 \\ \times 23 \\ \hline 33 \\ 220 \\ \hline 253 \end{array}$	$\begin{array}{r} 11 \\ \times 23 \\ \hline 33 \\ 220 \\ \hline 253 \end{array}$	The boy scouts eat twenty-three grapes each on their campout. How many total grapes did the troop of eleven boys eat?

If one student has placed a particular card, challenge their partner to provide an explanation.

Maria placed this Lattice card with this Model. Martin, why has Maria placed it here?

If you find students have difficulty articulating their decisions, then you may want to use the questions from the *Common Issues* table to support your questioning.

If the whole class is struggling on the same issue, then you may want to write a couple of questions on the board and organize a whole class discussion.

Placing Card Sets D, E & F: Partial Products, Traditional Algorithm, & Word Problems

As students finish placing the Model, Distributive Property, and Lattice cards, hand out *Card Sets D, E & F: Partial Products, Traditional Algorithm & Word Problems*. These provide students with different ways of interpreting the situation.

Do not collect the card sets they have been using. An important part of this task is for students to make connections between all the different representations of multiplication problems.

As you monitor the work, listen to the discussion and help students to look for patterns and generalizations. Groups should have 8 different clusters of cards with 6 cards in each. The original cards show the correct matches on each row of the table they are originally arranged in.

Sharing work (10 minutes)

When students get as far as they can with matching cards, ask one student from each group to visit another group's work. Students remaining at their desk should explain their reasoning for the matched cards on their own desk.

If you are staying at your desk, be ready to explain the reasons for your group's matches.

If you are visiting another group, make note of your card placements on a piece of paper. Go to another group's desk and check to see which matches are different from your own.

If there are differences, ask for an explanation. If you still don't agree, explain your own thinking.

When you return to your own desk, you need to consider, as a group, whether to make any changes to your work.

Students may now want to make changes.

Improve individual solutions to the assessment task (10 minutes)

Return to the students their original assessment, *Multiplication Strategies & Representations* as well as a second blank copy of the task.

Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to improve your work.

If you have not added questions to individual pieces of work then write your list of questions on the board.

Students should select from this list only the questions appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson, or for homework.

Solutions

Assessment Task: *Multiplication Strategies & Representations*

Question 1: $28 \times 17 = 476$ Students show work, but strategies may vary.

Question 2: Each of the responses from Sam, Julie, Pete, Lisa, & Fred are all correct. Sam used the traditional method, Julie used lattice method, Pete used clustering with the distributive property & some partial products, Lisa used a box method that shows the partial products in the area model, Fred drew a base-10 block area model representation. Be sure that the responses have correct interpretations of each model, but answers may vary in the way each is described.

Question 3: Student should state which person's strategy most closely matched their own work in question #1. They should then use a different strategy to solve $39 \times 14 = 546$ correctly.

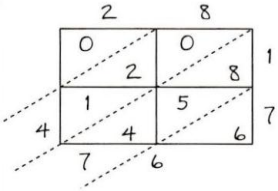
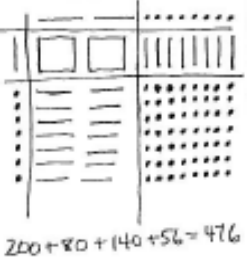
These materials were adapted from *Everyday Mathematics*, *Uncovering Student Misconceptions in Mathematics*, and the *National Library of Virtual Manipulatives*.

When teaching these multiplication and division strategies, *Teaching Student Centered Mathematics* by Van de Walle will be a useful resource.

Multiplication Strategies & Representations Task

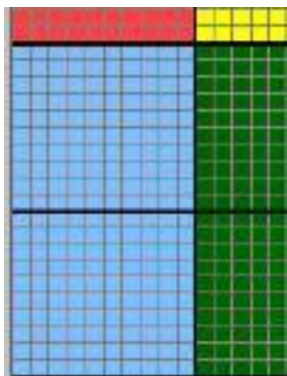
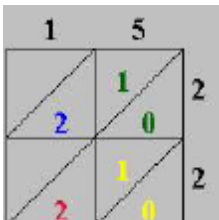

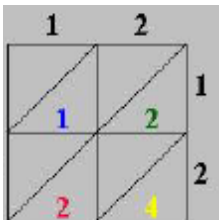
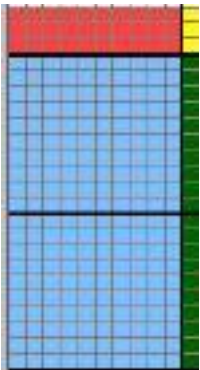
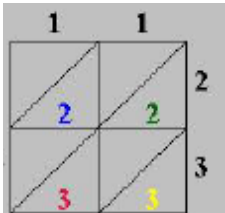
1.) Multiply 28 by 17 and show your work:

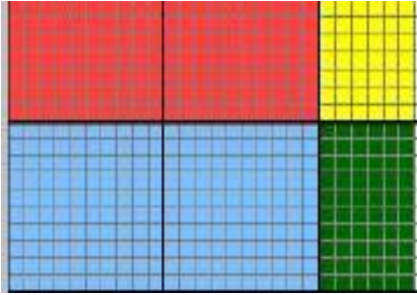
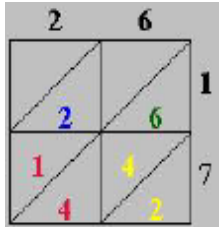
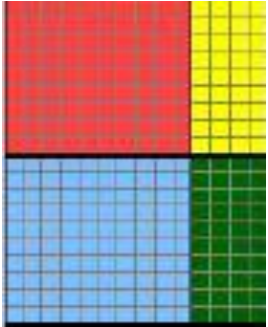
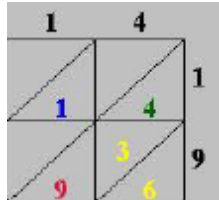
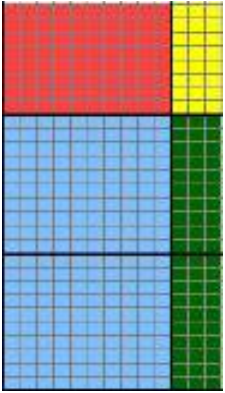
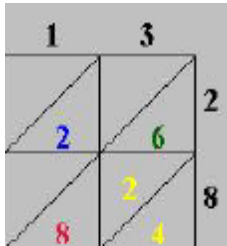
2.) Sam, Julie, Pete, Lisa, & Fred each multiplied 28 by 17. Below each method indicate if the work is correct and then explain whether that method makes sense mathematically or not.

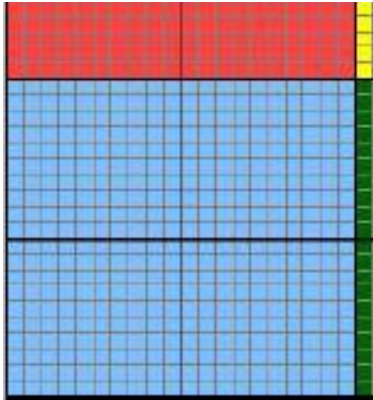
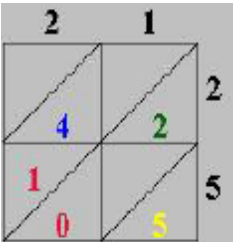
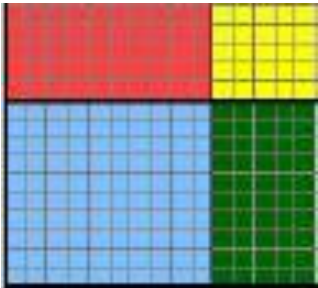
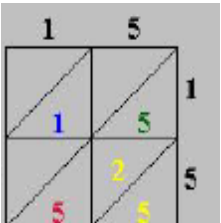
<i>Sam</i>	<i>Julie</i>	<i>Pete</i>	<i>Lisa</i>	<i>Fred</i>
$\begin{array}{r} ^5 28 \\ 17 \\ \hline 196 \\ + 280 \\ \hline 476 \end{array}$		$\begin{aligned} 28 \times 10 &= 280 \\ 28 \times 5 &= 140 \\ 28 \times 2 &= 56 \\ 280 + 140 + 56 &= 476 \end{aligned}$	$\begin{array}{cc} 20 & 8 \\ 10 & \begin{array}{ c c } \hline 200 & 80 \\ \hline \end{array} \\ 7 & \begin{array}{ c c } \hline 140 & 56 \\ \hline \end{array} \\ \hline 200 + 80 + 140 + 56 = 476 \end{array}$	
Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:	Check one: <input type="checkbox"/> correct <input type="checkbox"/> incorrect Explain why:

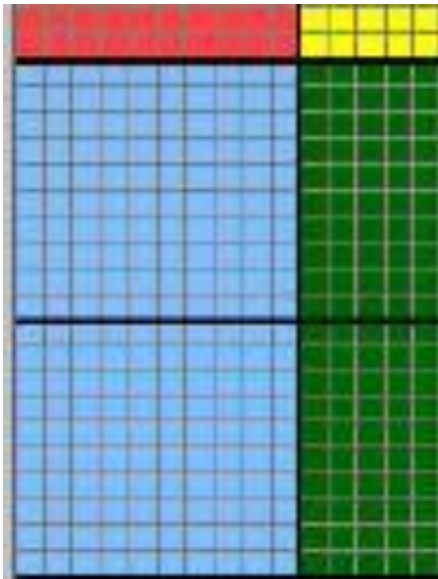
3.) Which method most closely matches how you solved the original problem? _____
Choose a different method than what you used in #1 to multiply 39 by 14.
Show your work below:

Multiplication Representations & Strategies CARDS

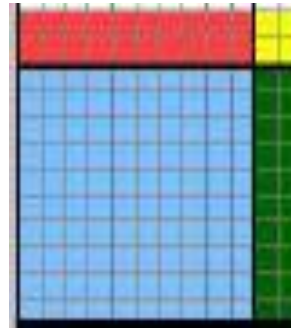
Model Card Set A	Lattice Card Set B	Distributive Property Card Set C	Partial Products Card Set D	Traditional Card Set E	Problem Card Set F
		$(20 + 2) \times (10 + 5) =$ $20 \times 10 + 2 \times 10 + 20 \times 5 + 2 \times 5 =$	$\begin{array}{r} 15 \\ \times 22 \\ \hline 30 \\ 300 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ \times 22 \\ \hline 30 \\ 300 \\ \hline \end{array}$	Each pack of baseball cards has fifteen cards. How many cards are in twenty-two packs?
		$(10 + 2) \times (10 + 2) =$ $10 \times 10 + 2 \times 10 + 10 \times 2 + 2 \times 2 =$	$\begin{array}{r} 12 \\ \times 12 \\ \hline 24 \\ 120 \\ \hline \end{array}$	$\begin{array}{r} 12 \\ \times 12 \\ \hline 24 \\ 120 \\ \hline \end{array}$	How many eggs are in twelve dozen?
		$(20 + 3) \times (10 + 1) =$ $20 \times 10 + 3 \times 10 + 20 \times 1 + 3 \times 1 =$	$\begin{array}{r} 11 \\ \times 23 \\ \hline 33 \\ 220 \\ \hline \end{array}$	$\begin{array}{r} 11 \\ \times 23 \\ \hline 33 \\ 220 \\ \hline \end{array}$	The boy scouts eat twenty-three grapes each on their campout. How many total grapes did the troop of eleven boys eat?

Model Card Set A	Lattice Card Set B	Distributive Property Card Set C	Partial Products Card Set D	Traditional Card Set E	Problem Card Set F
		$(10 + 7) \times (20 + 6) =$ $10 \times 20 + 7 \times 20 + 10 \times 6 + 7 \times 6 =$	$\begin{array}{r} 26 \\ \times 17 \\ \hline 42 \\ 140 \\ \hline 200 \end{array}$	$\begin{array}{r} 26 \\ \times 17 \\ \hline 182 \\ 260 \\ \hline \end{array}$	The deck Scott is building needs twenty-six boards and each board needs seventeen nails. How many nails does Scott need to buy?
		$(10 + 9) \times (10 + 4) =$ $10 \times 10 + 9 \times 10 + 10 \times 4 + 9 \times 4 =$	$\begin{array}{r} 14 \\ \times 19 \\ \hline 36 \\ 90 \\ \hline 100 \end{array}$	$\begin{array}{r} 14 \\ \times 19 \\ \hline 126 \\ 140 \\ \hline \end{array}$	An opossum sleeps an average of nineteen hours per day. How many hours does it sleep in a 2-week time period?
		$(20 + 8) \times (10 + 3) =$ $20 \times 10 + 8 \times 10 + 20 \times 3 + 8 \times 3 =$	$\begin{array}{r} 13 \\ \times 28 \\ \hline 24 \\ 80 \\ \hline 200 \end{array}$	$\begin{array}{r} 13 \\ \times 28 \\ \hline 104 \\ 260 \\ \hline \end{array}$	Cam bought thirteen different colored folders and each had twenty-eight dots. How many total dots are on her folders?

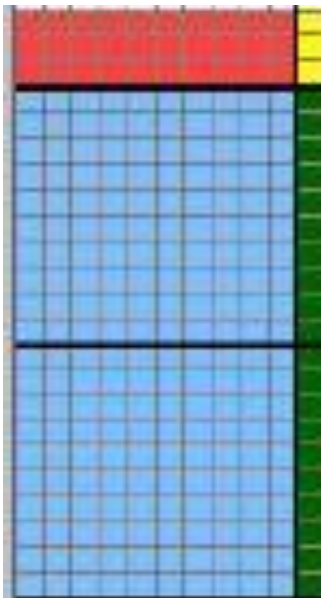
Model Card Set A	Lattice Card Set B	Distributive Property Card Set C	Partial Products Card Set D	Traditional Card Set E	Problem Card Set F
		$(20 + 5) \times (20 + 1) =$ $20 \times 20 + 5 \times 20 + 20 \times 1 + 5 \times 1 =$	$\begin{array}{r} 21 \\ \times 25 \\ \hline 105 \\ 420 \\ \hline \end{array}$	$\begin{array}{r} 21 \\ \times 25 \\ \hline 105 \\ 420 \\ \hline \end{array}$	Bags of reese's cups have twenty-one individually wrapped peanut butter cups. How many cups are in twenty-five bags?
		$(10 + 5) \times (10 + 5) =$ $10 \times 10 + 5 \times 10 + 10 \times 5 + 5 \times 5 =$	$\begin{array}{r} 15 \\ \times 15 \\ \hline 75 \\ 150 \\ \hline \end{array}$	$\begin{array}{r} 15 \\ \times 15 \\ \hline 75 \\ 150 \\ \hline \end{array}$	The zoo has fifteen monkeys who eat fifteen bananas each day. How many bananas does the need each day for the monkeys?



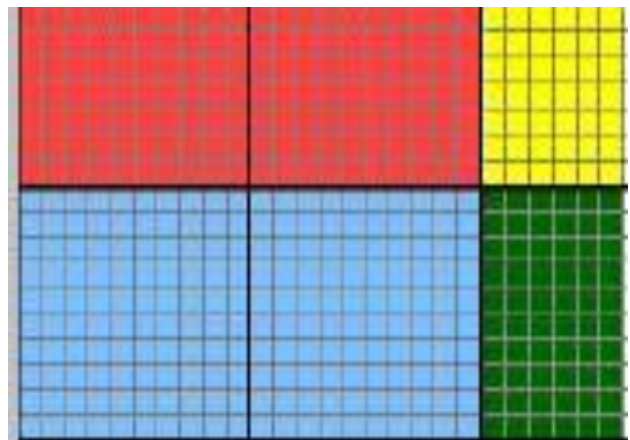
Card Set A



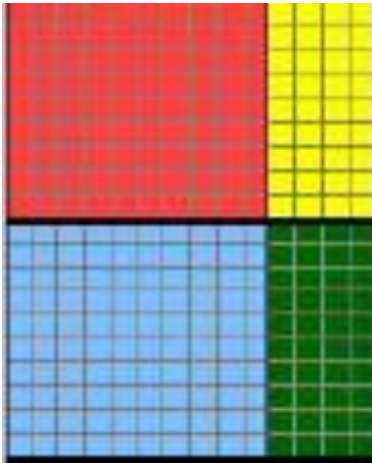
Card Set A



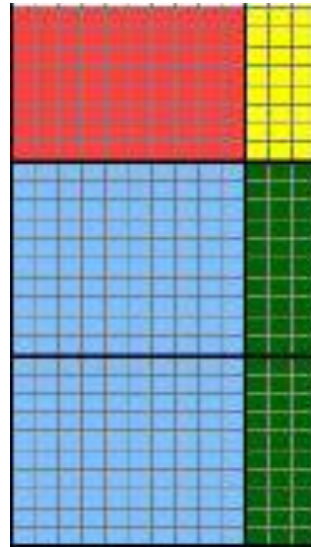
Card Set A



Card Set A



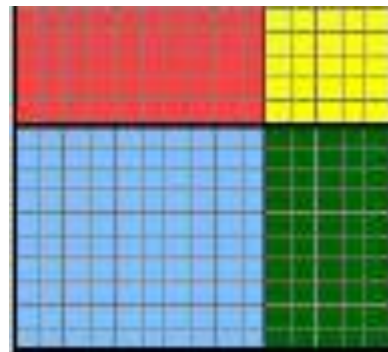
Card Set A



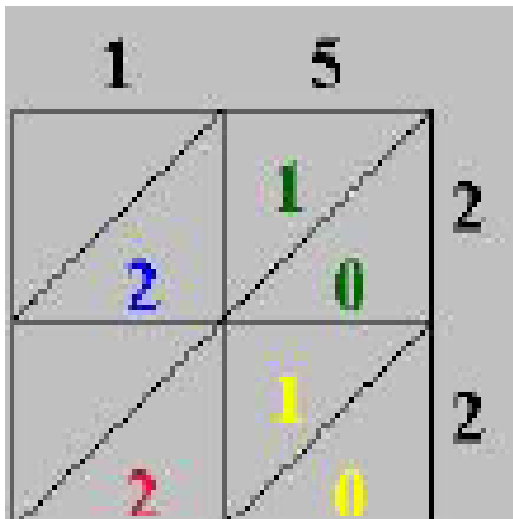
Card Set A



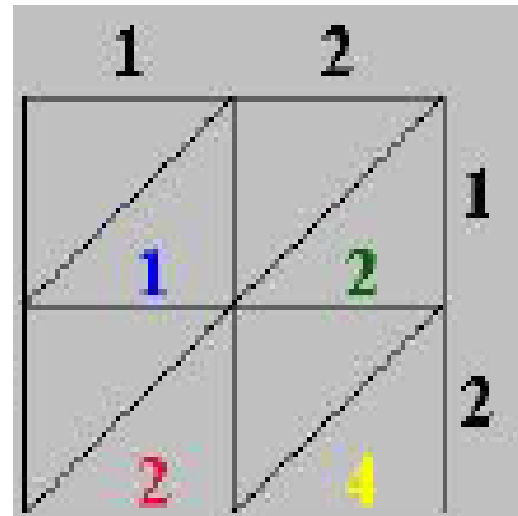
Card Set A



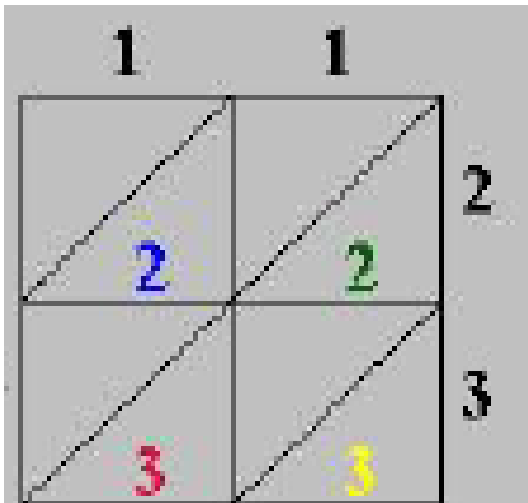
Card Set A



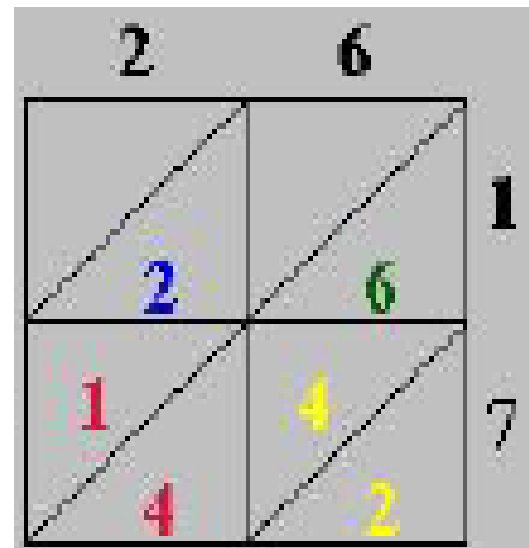
Card Set B



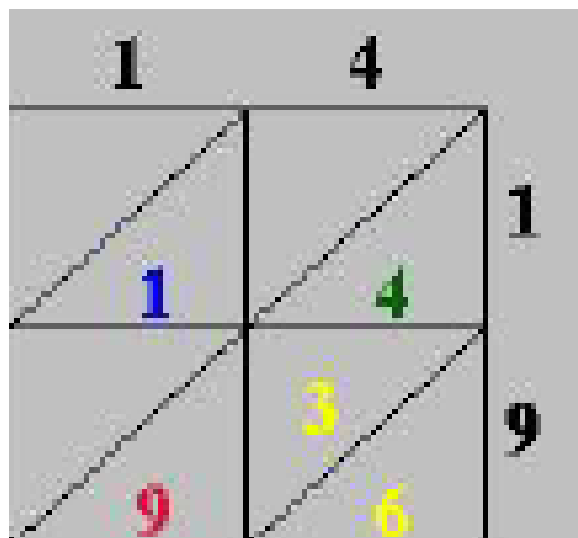
Card Set B



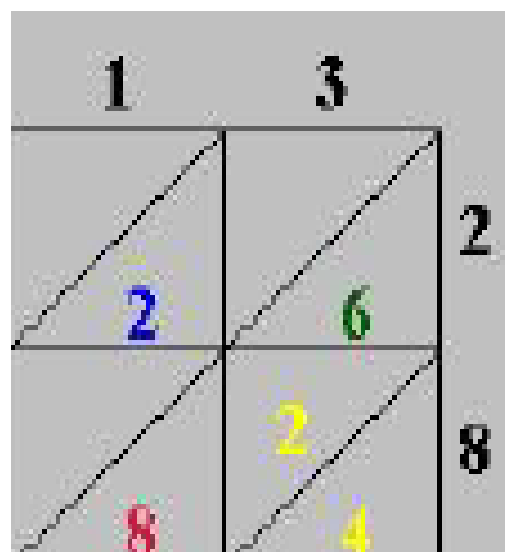
Card Set B



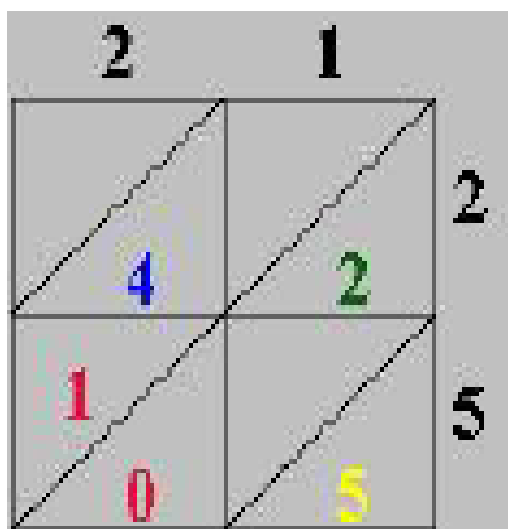
Card Set B



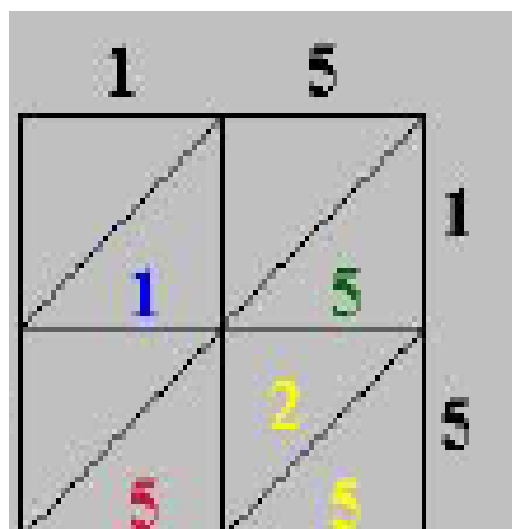
Card Set B



Card Set B



Card Set B



Card Set B

$$(20 + 2) \times (10 + 5) =$$

$$20 \times 10 + 2 \times 10 + 20 \times 5 + 2 \times 5 =$$

Card Set C

$$(20 + 3) \times (10 + 1) =$$

$$20 \times 10 + 3 \times 10 + 20 \times 1 + 3 \times 1 =$$

Card Set C

$$(10 + 9) \times (10 + 4) =$$

$$10 \times 10 + 9 \times 10 + 10 \times 4 + 9 \times 4 =$$

Card Set C

$$(20 + 5) \times (20 + 1) =$$

$$20 \times 20 + 5 \times 20 + 20 \times 1 + 5 \times 1 =$$

Card Set C

$$(10 + 2) \times (10 + 2) =$$

$$10 \times 10 + 2 \times 10 + 10 \times 2 + 2 \times 2 =$$

Card Set C

$$(10 + 7) \times (20 + 6) =$$

$$10 \times 20 + 7 \times 20 + 10 \times 6 + 7 \times 6 =$$

Card Set C

$$(20 + 8) \times (10 + 3) =$$

$$20 \times 10 + 8 \times 10 + 20 \times 3 + 8 \times 3 =$$

Card Set C

$$(10 + 5) \times (10 + 5) =$$

$$10 \times 10 + 5 \times 10 + 10 \times 5 + 5 \times 5 =$$

Card Set C

$$\begin{array}{r}
 15 \\
 \times 22 \\
 \hline
 10 \\
 20 \\
 100 \\
 \underline{200}
 \end{array}$$

Card Set D

$$\begin{array}{r}
 12 \\
 \times 12 \\
 \hline
 4 \\
 20 \\
 20 \\
 \underline{100}
 \end{array}$$

Card Set D

$$\begin{array}{r}
 11 \\
 \times 23 \\
 \hline
 3 \\
 30 \\
 20 \\
 \underline{200}
 \end{array}$$

Card Set D

$$\begin{array}{r}
 26 \\
 \times 17 \\
 \hline
 42 \\
 140 \\
 60 \\
 \underline{200}
 \end{array}$$

Card Set D

$$\begin{array}{r} 14 \\ \times 19 \\ \hline \end{array}$$

36

90

40

100

Card Set D

$$\begin{array}{r} 13 \\ \times 28 \\ \hline \end{array}$$

24

80

60

200

Card Set D

$$\begin{array}{r} 21 \\ \times 25 \\ \hline \end{array}$$

5

100

20

400

Card Set D

$$\begin{array}{r} 15 \\ \times 15 \\ \hline \end{array}$$

25

50

50

100

Card Set D

$$\begin{array}{r} 26 \\ \times 17 \\ \hline 182 \\ 260 \\ \hline \end{array}$$

Card Set E

$$\begin{array}{r} 14 \\ \times 19 \\ \hline 126 \\ 140 \\ \hline \end{array}$$

Card Set E

$$\begin{array}{r} 15 \\ \times 22 \\ \hline 30 \\ 300 \\ \hline \end{array}$$

Card Set E

$$\begin{array}{r} 12 \\ \times 12 \\ \hline 24 \\ 120 \\ \hline \end{array}$$

Card Set E

$$\begin{array}{r} 11 \\ \times 23 \\ \hline 33 \\ 220 \\ \hline \end{array}$$

Card Set E

$$\begin{array}{r} 13 \\ \times 28 \\ \hline 104 \\ 260 \\ \hline \end{array}$$

Card Set E

$$\begin{array}{r} 21 \\ \times 25 \\ \hline 105 \\ 420 \\ \hline \end{array}$$

Card Set E

$$\begin{array}{r} 15 \\ \times 15 \\ \hline 75 \\ 150 \\ \hline \end{array}$$

Card Set E

The deck Scott is building needs twenty-six boards and each board needs seventeen nails. How many nails does Scott need to buy?

Card Set F

An opossum sleeps an average of nineteen hours per day. How many hours does it sleep in a 2-week time period?

Card Set F

Cam bought thirteen different colored folders and each had twenty-eight dots. How many total dots are on her folders?

Card Set F

Each pack of baseball cards has fifteen cards. How many cards are in twenty-two packs?

Card Set F

**How many eggs are
in twelve dozen?**

Card Set F

**The boy scouts eat
twenty-three
grapes each on
their campout.
How many total
grapes did the
troop of eleven
boys eat?**

Card Set F

**Bags of Reese's
cups have twenty-
one individually
wrapped peanut
butter cups. How
many cups are in
twenty-five bags?**

Card Set F

**The zoo has fifteen
monkeys who eat
fifteen bananas
each day. How
many bananas do
they need each day
for the monkeys?**

Card Set F